Topological Channels in Bilayer Graphene in Electric and Magnetic Fields

Andrew Jiao, Aaron Winn, Charlie Maier, Juan Velasco, Matt Walker, Jeffrey C.Y. Teo
Department of Physics, University of Virginia

Abstract

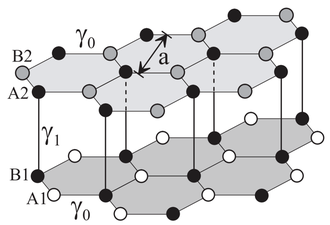
In recent years, bilayer graphene has attracted theoretical attention due to its extraordinary electronic properties. In particular, it has been recently predicted [1] and experimentally demonstrated [2] that bilayer graphene under an electric potential that changes sign at an interface induces exponentially localized chiral edge states. These edge states are robust against perturbation due to the topological properties of the 2D bulk. We confirm these results using the tight-binding model on a cylindrical geometry with periodic boundary conditions and obtain analytical solutions of the wavefunction using k·p expansion about the Dirac points. By applying an additional uniform magnetic field to this system, we show that the edge states deviate from the interface.

Tight-binding: cylindrical geometry

The AB-stacked bilayer graphene has a tight-binding Hamiltonian of the form

$$H = \sum_{\mathbf{R}\in\mathcal{L}} \gamma_0 (|\mathbf{R}, \circ_1\rangle + |\mathbf{R} - \mathbf{a}_2, \circ_1\rangle + |\mathbf{R} - \mathbf{a}_1 - \mathbf{a}_2, \circ_1\rangle) \langle \mathbf{R}, \bullet_1| + \gamma_1 |\mathbf{R}, \circ_2\rangle\langle \mathbf{R}, \bullet_1| + \text{h.c.}$$

We neglect spin, next-nearest neighbor hopping, and other weaker tunneling processes. The energy dispersion is quadratic at the K and K' points (Figure 3), which is then gapped out in the presence of a perpendicular electric field. Applying an alternating electric field breaks translational symmetry in one direction. To avoid localized states in a ribbon geometry, we employ periodic boundary conditions. The resulting cylinder has two interfaces where the electric potential changes sign and where edge states are localized (Figure 4). To minimize finite size effects, our calculations use a sufficiently large unit cell.



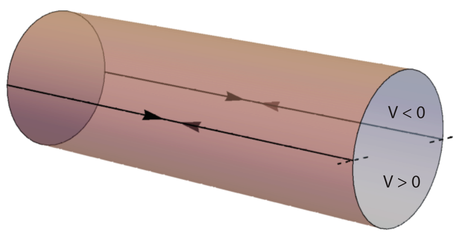


Figure 1 (left). AB stacked bilayer graphene. [3] $\gamma_0 = 3.16$ eV, $\gamma_1 = 0.381$ eV, a = 2.46 Å. [4] **Figure 2 (right).** Cylindrical geometry, showing the two conducting channels where counterpropagating modes are localized.

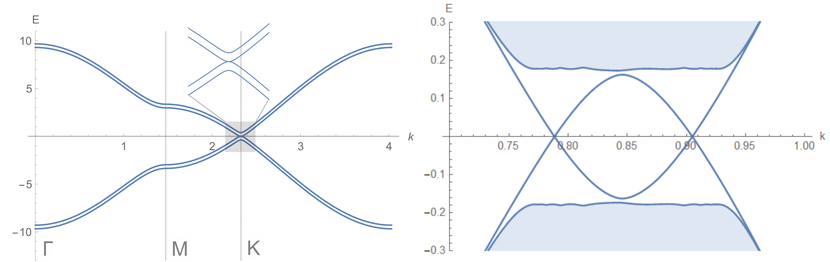


Figure 3 (left). Energy dispersion of bilayer graphene about the high symmetry points in the Brillouin zone, showing quadratic band touching at the K point. **Figure 4 (right).** Energy dispersion (eV vs. Å^{-1}) of the edge states in the lattice model at the K point at $V_0 = 1$. The counterpropagating modes reside on different interfaces.

Continuum limit

The Hamiltonian

$$H(x, k_y) = \begin{pmatrix} V(x) & v\pi & 0 & \gamma_1 \\ v\pi^\dagger & V(x) & 0 & 0 \\ 0 & 0 & -V(x) & v\pi \\ \gamma_1 & 0 & v\pi^\dagger & -V(x) \end{pmatrix} \qquad \pi = -i\hbar\frac{\partial}{\partial x} + i\hbar k_y \qquad v = \frac{\sqrt{3}}{2\hbar} a\gamma_0 \approx 0.003c$$

describes the first-order expansion at the K point in the Brillouin zone. We set $V(x) = V_0\,\text{sgn}(x)/2$ in our calculations. The zero-energy eigenstates that solve this system are found to satisfy

$$k_y^{\pm} = \pm\frac{V_0}{4\hbar v}\sqrt{1 + \sqrt{1 + 8\left(\frac{\gamma_1}{V_0}\right)^2}} \qquad \lim_{V_0\to\infty} \Delta x_\psi = \frac{\hbar v}{\gamma_1} \approx 17.7 \text{ Å}$$

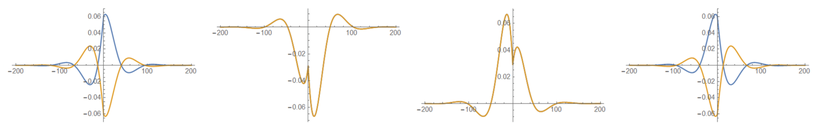


Figure 5. Wavefunction components for $V_0 = 1$. Blue (orange) denotes the real (imaginary) part. The second and third components have overlapping real and imaginary parts.

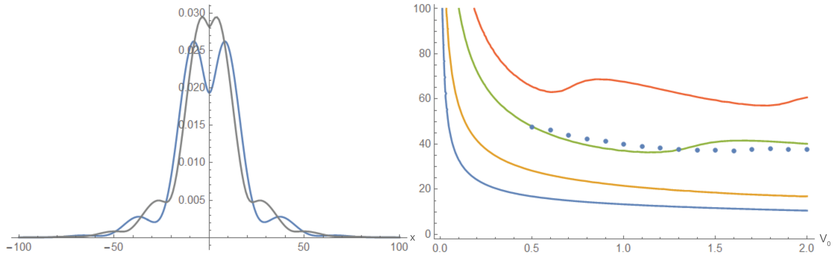


Figure 6 (left). Comparison of $|\psi|^2$ between lattice (gray) and continuum (blue) models at $V_0 = 1$. **Figure 7 (right).** Width of $|\psi|^2$ (Å) as a function of potential strength (eV) for probability cutoffs of 0.5 (blue), 0.75 (orange), 0.95 (green), and 0.99 (red), with tight-binding calculations superimposed for $0.5 < V_0 < 2$.

Landau levels

Bilayer graphene possesses a Landau quantization under the presence of a strong magnetic field. Focusing on the low-lying energy spectrum, we realize a two-band effective Hamiltonian

$$H = \begin{pmatrix} 0 & v\pi & 0 & \gamma_1 \\ v\pi^\dagger & 0 & 0 & 0 \\ 0 & 0 & 0 & v\pi \\ \gamma_1 & 0 & v\pi^\dagger & 0 \end{pmatrix} \xrightarrow[E\ll\gamma_1,\gamma_0]{} H_{\text{eff}} = -\frac{v^2}{\gamma_1}\begin{pmatrix} 0 & \pi^2 \\ (\pi^\dagger)^2 & 0 \end{pmatrix}$$

and assume that the wavefunctions are shifted harmonic oscillator eigenstates

$$\phi_n(x, k_y) \propto e^{-\tilde{x}^2/2} H_n(\tilde{x}), \quad \tilde{x} = \frac{x}{\ell_B} - \ell_B k_y, \quad \ell_B = \sqrt{\frac{\hbar}{eB}}$$

With appropriate use of ladder operators, we rederived the spectrum

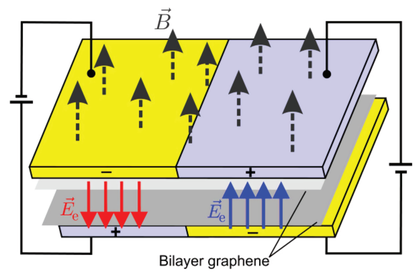
$$E_n = \text{sgn}(n)\frac{2\hbar^2 v^2}{\gamma_1 \ell_B^2}\sqrt{|n|(|n|-1)}$$

Strong electric and magnetic fields

Finally, we add back the electric field so that both fields are perpendicular to the bilayer graphene sheet (Figure 8). The magnetic field strength takes on the form

$$B = \frac{\phi_0}{\text{Area}}\frac{p}{q}$$

where p/q denotes the filling fraction of the magnetic flux quantum ϕ_0 through a single hexagon plaquette. In general, there will be a delta magnetic flux spike at one interface, but we take advantage of the cylindrical geometry to isolate edge states on the interface with uniform flux.



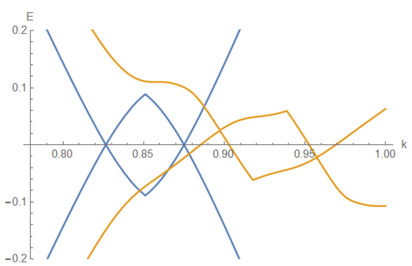
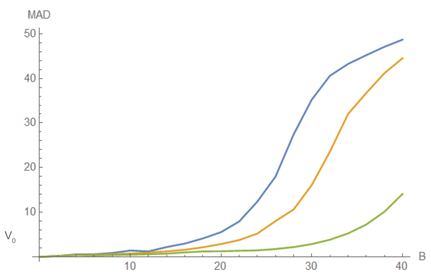


Figure 8 (left). Bilayer graphene with an electric field that changes sign at an interface and a uniform magnetic field. [5] Both fields are perpendicular to the plane of the bilayer. **Figure 9 (right).** Energy dispersion of the edge states at $V_0 = 0.3$ for B = 0 (blue) and B = 24 (orange), showing the shifting K point.



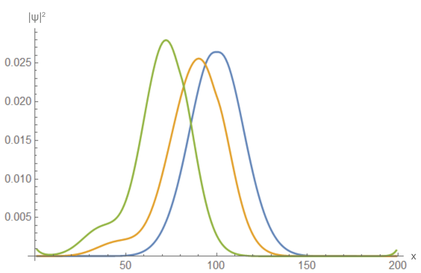


Figure 10 (left). Mean absolute deviation of $|\psi|^2$ as a function of field strength (T) at $V_0 = 0.3$ (blue), 0.5 (orange), and 1 (green). **Figure 11 (right).** Deviating $|\psi|^2$ at $V_0 = 0.3$ for B = 0 (blue), 24 (orange), and 32 (green).

Conclusion

We have shown the existence of exponentially localized chiral edge states using both the lattice and continuum models. We were able to isolate those edge states that reside on the interface located far away from the delta magnetic flux spike. The characteristic width and mean absolute deviation from the interface were found as a function of electric and magnetic field strength. These parameters are important for quantum transport systems [6] that may be used to engineer ultra low-power electronics.

References

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